Supplementary File

Time-Driven Priority Router Implementation: Analysis and Experiments

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APPENDINX A: THEOREM PROOFS

Appendix A.1: Proof of Theorem 1

Theorem 1. *Sufficient and necessary condition for* $N_{i-1} = \tilde{N}_{i-1}$ *for any packet transmitted in TF* N_{i-1} *is that* α *is an arbitrarily small time interval.*

Proof. Noting that $T = t + P$, where t is the transmission time of a packet at node $i-1$, and deriving $\tilde{T} - \tilde{D}$ from (5), (6) can be rewritten as

$$\left\lfloor \frac{t}{\hat{T}_f} \right\rfloor = \left\lfloor \frac{t+\alpha}{\hat{T}_f} \right\rfloor. \qquad \text{(A.1)}$$

In order to prove Theorem 1 we need to find all the values of α that satisfy (A.1) $\forall t \in [N_{i-1} \cdot \hat{T}_f, (N_{i-1}+1) \cdot \hat{T}_f)$. Let

- $t_m = \min t = N_{i-1} \cdot \hat{T}_f$,
- $t_M = \max t$,
- $l(t)$ and $r(t,\alpha)$ be the left and the right member of (A.1), respectively,
- $A_m \subseteq \mathbb{R}$ such that

$$\begin{cases} l(t_m) = r(t_m,\alpha), \forall \alpha \in A_m \\ l(t_m) \neq r(t_m,\alpha), \forall \alpha \notin A_m \end{cases}, \qquad \text{(A.2)}$$

- $A_M \subseteq \mathbb{R}$ such that

$$\begin{cases} l(t_M) = r(t_M,\alpha), \forall \alpha \in A_M \\ l(t_M) \neq r(t_M,\alpha), \forall \alpha \notin A_M \end{cases}, \qquad \text{(A.3)}$$

Lemma 1. $\alpha \in A = A_m \cap A_M$ *is the necessary and sufficient condition for* $N_{i-1} = \tilde{N}_{i-1}$.

Proof of Sufficiency. Given that $l(t)$ and $r(t,\alpha)$ are monotonic increasing functions of t and α, values of α that satisfy (A.1) for both t_m and t_M, satisfy (A.1) also for any intermediate value of t, $t_m \le t \le t_M$. A is a set of solutions that satisfy (A.1) for any value of t, $t_m \le t \le t_M$.

Proof of Necessity. By contradiction, suppose $\alpha_0 \notin A$ satisfies (A.1), i.e., $l(t) = r(t,\alpha_0), t \neq t_m, t \neq t_M$. In order to be a solution for $N_{i-1} = \tilde{N}_{i-1}$, α_0 must satisfy (A.1) $\forall t \in [N_{i-1} \cdot \hat{T}_f, (N_{i-1}+1) \cdot \hat{T}_f)$, i.e., also in t_m and t_M. This contradicts the assumption $\alpha_0 \notin A$. Consequently, $\alpha \in A$ is a necessary condition for $N_{i-1} = \tilde{N}_{i-1}$.

□

Theorem 1 can be derived by evaluating A, which is done in the following. By substituting t_m in (A.1) we obtain

$$N_{i-1} = N_{i-1} + \left\lfloor \frac{\alpha}{\hat{T}_f} \right\rfloor \qquad \text{(A.4)}$$

which implies $A_m = [0, \hat{T}_f)$. Let us also consider that a packet could be sent an arbitrarily small interval $J > 0$ before the end of its forwarding TF N_{i-1}, i.e., $t_M = (N_{i-1}+1) \cdot \hat{T}_f - J$, $0 < J << \hat{T}_f$. This value, can be substituted in (A.1), from which we obtain $A_M = [J - \hat{T}_f, J)$, from which it can be concluded that $\tilde{N}_{i-1} = N_{i-1}$ for $0 \le \alpha < J$, $0 < J << \hat{T}_f$, i.e., α must be an arbitrarily small positive number.

□

Appendix A.2: Proof fo Theorem 2

Theorem 2. *Given a guard time band of duration* $\hat{\gamma}$*, the necessary and sufficient condition for* $N_{i-1} = \tilde{N}_{i-1}$ *for any packet transmitted in TF* N_{i-1} *is that* $-\hat{\gamma} \le \alpha < \hat{\gamma}$.

Proof. The proof can be carried out from Lemma 1, following the same steps taken for the proof of Theorem 1 and taking into account that no transmission can occur just after the beginning and just before the end of the TF, i.e., within the guard time bands. This implies $t_m = N_{i-1} \cdot \hat{T}_f + \hat{\gamma}$ and $t_M = (N_{i-1}+1) \cdot \hat{T}_f - \hat{\gamma}$. Specifically, A_m can be obtained by solving

$$\left\lfloor \frac{\hat{\gamma}}{\hat{T}_f} \right\rfloor = \left\lfloor \frac{\hat{\gamma}+\alpha}{\hat{T}_f} \right\rfloor, \qquad \text{(A.5)}$$

which, since $\hat{\gamma} << \hat{T}_f$, provides $A_m = [-\hat{\gamma}, \hat{T}_f - \hat{\gamma})$. AM is instead devised from

$$\left\lfloor \frac{(N_{i-1}+1) \cdot \hat{T}_f - \hat{\gamma}}{\hat{T}_f} \right\rfloor = \left\lfloor \frac{(N_{i-1}+1) \cdot \hat{T}_f - \hat{\gamma} + \alpha}{\hat{T}_f} \right\rfloor \qquad \text{(A.6)}$$

as $A_M = [\hat{\gamma} - \hat{T}_f, \hat{\gamma})$.

From $A_m \cap A_M$ it can be concluded that when a guard time band of duration $\hat{\gamma}$ is deployed, $N_{i-1} = \tilde{N}_{i-1}$ for $-\hat{\gamma} \le \alpha < \hat{\gamma}$.

□

Appendix A.3: Proof of Theorem 3

Theorem 3. *Necessary and sufficient condition on the forwarding delay* $d_{(i-1)i}$ *(measured in TFs) to guarantee correct pipeline forwarding operation is:*

$$d_{(i-1)i} > \left[\frac{E_R + E_T + T_T + \hat{P} + P + R_R + \hat{T}e + M}{\hat{T}_f}\right] + 1 . \quad (A.7)$$

Proof. Let Tou_i denote the time at which the transmission of the first packet is scheduled at node i for a forwarding TF N_i, and $Tinb_i$ be the time at which a packet scheduled for TF N_i enters the the output buffer of node i. Since $N_i \cdot \hat{T}_f$ is the time at which N_i would begin at the node if an ideal CTR were used, we can write

$$Tou_i = t_1 + e_R \quad (A.8)$$

and

$$Tinb_i = t_2 + e_T + t_T + \hat{P} + p + r_R + Te, \quad 0 \le t_2 < \hat{T}_f , (A.9)$$

where $t_1 \in [N_i \cdot \hat{T}_f, (N_i+1)\cdot\hat{T}_f)$ and $t_2 \in [N_{i-1}\cdot\hat{T}_f, (N_{i-1}+1)\cdot\hat{T}_f)$ are the times at which the transmission of the packet starts at node i and finishes[1] at node $i-1$, respectively. As stated by Rule 1 in Section 2.1, the condition for correct pipeline forwarding operation is $Tou_i > Tinb_i$. This has to hold for every value of latencies and inaccuracies, and for any packet transmitted during TF N_{i-1}, specifically for the worst case. Hence,

$$\min_{t_1 = N_i \cdot \hat{T}_f} (Tou_i) > \max_{t_2 = (N_{i-1}+1)\cdot\hat{T}_f - J} (Tinb_i) , \quad (A.10)$$

where $J > 0$ is an arbitrarily small number. Given the definition of the involved accuracies, latencies and delays provided in Section 4.1, the worst case condition can be expressed as:

$$N_i \cdot \hat{T}_f - E_R > (N_{i-1}+1)\cdot\hat{T}_f - J + E_T + T_T + \hat{P} + P + R_R + \hat{T}e + M . \quad (A.11)$$

Devising the forwarding delay $d_{(i-1)i}$, that is by definition the integer number of TFs between the forwarding TFs in subsequent nodes, and considering that $x > y - J \Leftrightarrow x \ge y$ for $J > 0$ arbitrarily small, we obtain

$$d_{(i-1)i} = N_i - N_{i-1} \ge \left[\frac{E_R + E_T + T_T + \hat{P} + P + R_R + \hat{T}e + M}{\hat{T}_f}\right] + 1 . \quad (A.12)$$

□

Appendix A.4: Proof of Theorem 4

Theorem 4. *In a TDP node deploying the inaccuracy-tolerant pipeline forwarding operating mode where* $-E_T \le e_T \le E_T$ *and* $0 \le t_T^{ctr} \le T_T^{ctr}$ *(i.e.,* $-E_T \le y \le E_T + T_T^{ctr}$*), the time difference between the actual TF beginning and ideal TF beginning is bounded as:*

$$\Delta T_n^b = \left| T_n^b - \hat{T}_n^b \right| \le E_T + T_T^{ctr} \quad \forall n \quad (A.13)$$

Proof. For the sake of notation simplicity, we consider an infinite sequence of TFs (i.e., $\{n\}_{n=0}^{\infty}$) instead of repeating sequences of H TFs (i.e., $\{n \bmod H\}_{n=0}^{\infty}$). Furthermore, let:

- T_n^e be the time at which TF n ends if the described inaccuracy-tolerant operating mode is used, i.e., the time at which the transmission of the packets scheduled during TF n finishes;
- y_n be the overall transmitter inaccuracy affecting the generic TF n.

The effect of the transmitter inaccuracy on the end of a TF, hence on the beginning of the next one, is mitigated or even compensated by the former TF not being fully utilized. Consequently, the worst case from the point of view of the difference between the actual (i.e., when packet transmission starts) and the nominal beginning (i.e., according to the CTR) of a TF is when it is fully utilized, which is considered in this proof[2]. In such worst case,

$$T_n^e - T_n^b = \hat{T}_f, \ \forall n \ge 0 . \quad (A.14)$$

In order to prove that ΔT_n^b is bounded, it is first necessary to consider the relationship between the end (T_{n-1}^e) and the beginning (T_n^b) of two generic subsequent TFs. In the case of ideal transmitter (i.e., when $y_n = 0$),

$$T_n^b = T_{n-1}^e = \hat{T}_n^b . \quad (A.15)$$

If $y_n \neq 0$, $T_1^b = T_0^e + dy_0$, where

$$dy_0 = \begin{cases} y_1 - y_0 & y_1 > y_0 \\ 0 & \text{otherwise} \end{cases}$$

and $T_0^e = \hat{T}_0^b + \hat{T}_f + y_0$. Recursively,

$$T_n^b = \hat{T}_0^b + n \cdot \hat{T}_f + y_0 + \sum_{j=0}^{n-1} dy_j , \quad (A.16)$$

where

$$dy_j = \begin{cases} y_j - \left(y_0 + \sum_{k=0}^{j-1} dy_k \right) & y_j > y_0 + \sum_{k=0}^{j-1} dy_k \\ 0 & \text{otherwise} \end{cases} . \quad (A.17)$$

From (A.15), considering that $\hat{T}_n^b = \hat{T}_0^b + n \cdot \hat{T}_f$ and that, by construction, $y_0 + \sum_{j=1}^{n} dy_j \ge 0$, we can derive

$$\Delta T_n^b = \left| T_n^b - \hat{T}_n^b \right| = y_0 + \sum_{j=0}^{n-1} dy_j . \quad (A.18)$$

ΔT_n^b is limited as there cannot exist any $\bar{n}$ such that $\Delta T_{\bar{n}}^b = y_0 + \sum_{k=0}^{\bar{n}-1} dy_k > E_T + T_T^{ctr}$. In fact, this could be re-written as $y_0 + \sum_{k=0}^{\bar{n}-2} dy_k + dy_{\bar{n}-1}$, where, according to (A.17), $dy_{\bar{n}-1}$ can have one of the two following values:

1. $dy_{\bar{n}-1} = y_{\bar{n}-1} - \left(y_0 + \sum_{k=0}^{\bar{n}-2} dy_k \right)$, i.e., $\Delta T_{\bar{n}}^b = y_{\bar{n}-1} > E_T + T_T^{ctr}$, which conflicts with the definition of y itself.
2. $dy_{\bar{n}-1} = 0$, which implies that $\Delta T_{\bar{n}}^b = y_0 + \sum_{k=0}^{\bar{n}-2} dy_k > y_0 + \sum_{k=0}^{\bar{n}-3} dy_k + dy_{\bar{n}-2}$.

Similarly, $dy_{\bar{n}-2}$ can have one of the two above values and the reasoning can be iterated until either (*i*) a $j \le \bar{n}$ is found such that $dy_{\bar{n}-j} \neq 0$ and case 1 above applies, or (*ii*) we obtain $\Delta T_{\bar{n}}^b = y_{\bar{n}-1} > E_T + T_T^{ctr}$ due to the fact that $dy_j = 0 \ \forall j \in [0, \bar{n}-1]$, which however con-

[1] In Theorem 3 we assume that a packet is transmitted completely during its forwarding TF. Pipeline forwarding operation does not impose this and Theorem 3 can be easily generalized to encompass the case in which the transmission of a packet ends after the end of the forwarding TF.

[2] This reservation also includes the bandwidth waste due to τ_T^{tx}.

flicts with the definition of y itself. Consequently,

$$\Delta T_n^b = \left| T_n^b - \hat{T}_n^b \right| \le E_T + T_T^{ctr}, \forall n \ge 0, \quad \text{(A.19)}$$

i.e., the inaccuracy in the beginning of any TF n is bounded by $E_T + T_T^{ctr}$.

□